On the Problem of FSM Networks Power Estimation Capability

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Abstract. We propose a power estimation technique for controllers that operates at the register-transfer level to provide early warning of a power problems. Our estimator is based on the use of entropy as a measure of the average activity in the final implementation of a circuit given FSM network description. FSM networks are constructed from a partition of a initial FSM state transition graph (STG). The technique has been implemented and tested on a variety of benchmarks.

1. Introduction

With the increasing of portable computing devices the importance of the power constraints during the design of digital devices has continuously increased in the past years. The digital circuits are usually represented as a combination of a data path which performs operations and a control part which controls the execution in the data path. A controller is the most important block of any discrete device which controls all information streams.

The controller synthesis is one of the important fields of the research in the design of digital circuits. They are mostly modelled as finite state machines (FSMs). The different attribute characteristics of a FSM such as area, power consumption, testability, etc. are the field for an optimisation efforts of designers and researchers. The design of a low power controller is a complicate problem because of a physical capacitance of controllers depends on the contents of control table, which are not known until run time. Complex VLSI and board-level circuits may be modelled as being composed of interacting finite state machines. These interactions primarily implement the exchange of information among component machines. The decomposition of a FSM into a network of interacting finite state machines is useful for both area and performance reasons, also it allows a drastic reduction in timing verification task complexity.

In our paper we address to the problem of the design of low power controllers represented in a form of the FSM network descriptions. It is also easy to see that interactions between component FSMs complicate the FSM network model as compared with the model of a finite state machine.

The FSM entropy function for power estimation in a FSM is studied in [1,2]. In our paper we consider the problem of entropy application for the power estimation in a FSM network. It gives opportunity to evaluate the influence of an information exchange between component FSMs on the value of a FSM network entropy function.

The sections of the paper are organised as follows. The method of FSM decomposition based on the partition of STG of FSM is presented in the section 2. In the section 3 we propose the models for an evaluation of the entropy value in FSM networks. Experimental results are in the section 4.

2. Decompositional Synthesis of Controller

Let FSM A be characterised by a 5-tuple (X,Y,S,δ,λ) where X,Y,S are the sets of primary inputs, primary outputs and internal states, δ and λ are the next state and output functions respectively. There are the number of many decompositions of the same FSM A therefore it may be realised by different FSM networks. Usually the following requirements are satisfied for a FSM decomposition.

1. Component FSM is the FSM of an uniquely determined type.
2. The number of connections between component FSMs must be minimum.

3. The number of input, output channels and states in component FSM are restricted by some parameters.

It is known two basic approaches of a FSM decomposition. The first one is based on the algebra Hartmanis-Stearns [3] where the set of orthogonal partitions is applied for decomposition of original FSM A into FSM network N. The controllers of the real complexity are modelled by FSMs that have large numbers of inputs, outputs and states. Because of the problem of the search of an orthogonal partition set is a very complicate one the application of algebraic approaches for FSM decomposition is reduced to the solution of a very great dimension problem and is unpractical in many cases. Besides, it is very difficult to find the problem solution that simultaneously satisfies to the conditions 1-3 have been formulated above.

s^t	s^{t+1}	$X(s^t,s^{t+1})$	$Y(s^t,s^{t+1})$	N
s_1	s_2	x_1x_3	y_6	1
	s_6	$\neg x_1x_3$	y_3y_5	2
	s_6	$\neg x_3$	y_3y_6	3
s_2	s_2	$x_1 x_2x_3 x_5$	y_4	4
	s_4	$x_1 x_2x_3 \neg x_5$	y_2y_4	5
	s_1	$x_1\neg x_2x_3$	$y_2y_4y_5$	6
	s_3	$\neg x_1\neg x_2x_3$	y_5	7
	s_5	$\neg x_1 x_2x_3\neg x_5$	y_2y_5	8
	s_6	$\neg x_1x_2x_3 x_5$	$y_1y_4y_5$	9
	s_6	$x_2\neg x_3 x_5$	$y_1y_2y_4$	10
	s_5	$x_2\neg x_3\neg x_5$	y_1y_4	11
	s_3	x_2x_3	y_1y_5	12
s_3	s_3	x_2x_4	y_5y_6	13
	s_5	$x_2\neg x_4$	$y_4y_6y_7$	14
	s_7	$\neg x_2$	y_4y_5	15
s_4	s_1	x_1x_3	$y_1y_2y_5$	16
	s_3	$\neg x_1x_3$	y_5y_7	17
	s_3	$\neg x_3$	y_2y_7	18
s_5	s_6	x_2	y_2	19
	s_9	$\neg x_2$	y_2y_6	20
s_6	s_5	x_2x_5	y_4y_7	21
	s_6	$x_2\neg x_5$	y_1y_7	22
	s_8	$\neg x_2$	y_6y_7	23
s_7	s_2	1	y_1	24
s_8	s_2	1	y_1y_2	25
s_9	s_5	$x_2\neg x_5$	y_4y_7	26
	s_6	x_2x_5	y_1y_7	27
	s_7	$\neg x_2$	y_1	28

Fig. 1. Transition Table of FSM A

For a FSM decomposition we consider the approach [4] based on the construction of one partition set $\pi=\{S^1,S^2,\ldots,S^k\}$ of the state set S. The method of decomposition we illustrate by the example shown in Fig.1. We assume that $\pi=\{S^1,S^2,S^3\}$, where $S^1=\{s_1,s_2,s_4\}$, $S^2=\{s_6,s_7,s_8\}$, $S^3=\{s_3,s_5,s_9\}$. To the pair (S,π) let us put in accordance the FSM network with $|\pi|=3$ component FSMs $A^i=(T^i,X^i,Y^i,\delta^i,\lambda^i)$, i=1,2,3.

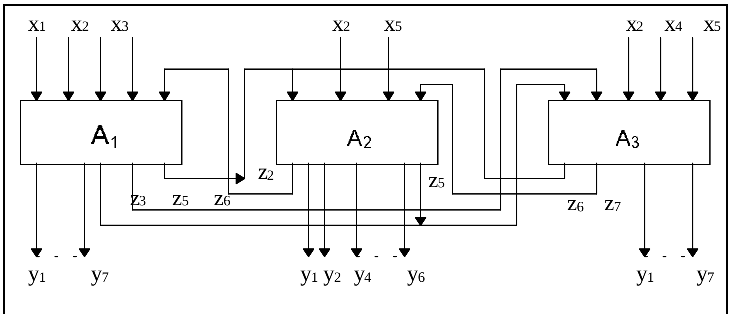

Fig.2. FSM network

s^t	s^{t+1}	$X(s^t,s^{t+1})$	$Y(s^t,s^{t+1})$	N
s_1	s_2	x_1x_3	y_6	1
	b_1	$\neg x_1x_3$	$y_3y_5z_6$	2
	b_1	$\neg x_3$	$y_3y_6\ z_6$	3
s_2	s_2	$x_1\ x_2x_3\ x_5$	y_4	4
	s_4	$x_1\ x_2x_3\ \neg x_5$	y_2y_4	5
	s_1	$x_1\ \neg x_2x_3$	$y_2y_4y_5$	6
	b_1	$\neg x_1\ \neg x_2x_3$	y_5z_3	7
	b_1	$\neg x_1\ x_2x_3\ \neg x_5$	$y_2y_5z_5$	8
	b_1	$\neg x_1\ x_2x_3\ x_5$	$y_1y_4y_5z_6$	9
	b_1	$x_2\neg x_3\ x_5$	$y_1y_2y_4z_6$	10
	b_1	$x_2\neg x_3\ \neg x_5$	$y_2y_4z_5$	11
	b_1	x_2x_3	$y_1y_5z_3$	12
s_4	s_1	x_1x_3	$y_1y_2y_5$	13
	b_1	$\neg x_1x_3$	$y_5y_7z_3$	14
	b_1	$\neg x_3$	$y_2y_7z_3$	15
b_1	s_2	z_2		16
	b_1	$\neg z_2$		17

Table1. Component FSM A^1

s^t	$s^{t+`1}$	$X(s^t,s^{t+1})$	$Y(s^t,s^{t+1})$	N
s_6	b_2	x_2x_5	$y_4y_7z_5$	1
	s_6	$x_2\neg x_5$	y_1y_7	2
	s_8	$\neg x_2$	y_6y_7	3
s_7	b_2	1	y_1z_2	4
s_8	b_2	1	$y_1y_2z_2$	5
b_2	s_6	z_6		6
	s_7	z_7		7
	b_2	$\neg z_6\neg z_7$		8

Table 2. Component FSM A^2

s^t	$s^{t+`1}$	$X(s^t,s^{t+1})$	$Y(s^t,s^{t+1})$	N
s_3	s_3	x_2x_4	y_5y_6	1
	s_5	$x_2\neg x_4$	$y_4\ y_6y_7$	2
	b_3	$\neg x_2$	$y_4\ y_5y_7$	3
s_5	b_3	x_2	y_2z_6	4
	s_9	$\neg x_2$	y_2y_6	5
s_9	s_5	$x_2\neg x_5$	y_4y_7	6
	b_3	x_2x_5	$y_1y_7z_6$	7
	b_3	$\neg x_2$	y_3z_7	8
b_3	s_3	z_3		9
	s_5	z_5		10
	b_3	$\neg z_3\neg z_5$		11

Table 3. Component FSM A^3

Component A^i is defined as follows:

1. The state set $T^i=S^i\cup\{b_i\}$, here S^i the state set that belongs to block S^i of partition π and b_i is an additional state . Thus, in our example the FSM network consists of three component FSMs (look Fig.2) A^1, A^2, A^3 for which the sets of states are determined by the blocks of partition π:

$T^1=\{s_1,s_2,s_4,b_1\}$, $T^2=\{s_6,s_7,s_8,b_2\}$, $T^3=\{s_3,s_5,s_9,b_3\}$.

2. The input channels of component A^i are defined as follows:

$X^i=X(S^i)\cup Z_x^{\ i}$, here $X(S^i)=\cup_{s\in S}{}^i\ X(s)$. $X(s)$ is the set of input variables in all conjunctions for the transition from s. $X(S^1)=\{x_1,x_2,x_3,x_4\}$, $X(S^2)=\{x_2,x_5\}$, $X(S^3)=\{x_2,x_4,x_5\}$. $Z_x^{\ i}=\{z_s\,|\,\delta(s_j,X_N=s_k,\ s_k\in S^i,\ s_j\notin S^i\}$. Here $X_N=X(s_j,s_k)$ is the input at the transition from s_j to s_k, $s_k\in S^i$. Thus, for each $s_k\in S^i$ let us put in accordance the additional input variable z_i in the component FSM A^i, if in the decomposed FSM A there is at least one transition to this state s_k from the state s_j not included in A^i. In our example $Z_x^{\ 1}=\{z_2\}$, $Z_x^{\ 2}=\{z_6,z_7\}$, $Z_x^{\ 3}=\{z_3,z_5\}$.

3.The output channels of component FSM A^i are defined as follows: $Y^i=Y(S^i)\cup Z_x^{\ i}$.

Here $Y(S^i)=\cup_{s\in S^i} Y(s)$, where $Y(s)$ is a set of output variables appearing in the column $Y(s^t,s^{t`1})$ at all transitions from the state s. In our example $Y(A^1)=\{y_1,y_2,y_3,y_4, y_5,y_6,y_7\}$, $Y(A^2)=\{y_1,y_2,y_4, y_5,y_6\}$, $Y(A^3)=\{y_1,y_2,y_3,y_4, y_5,y_6,y_7\}$.

Now let us define the set $Z_y^i=\{z_s \mid \delta(s_j,X_N)=s_k,\ s_j\in S^i,\ s_k\notin S^i\}$. So for each $s_k\notin S^i$ let us put in accordance the additional output variable z_s in the component A^i if in the FSM A there is at least one transition to this state s_k from the state s_j included in S^i. In our example $Z_y^1=\{z_3,z_5,z_6\}$, $Z_y^2=\{z_2,z_5\}$, $Z_y^3=\{z_6,z_7\}$.

4. Further we define a next state δ_i and output λ_i functions of component FSM A^i. In the FSM A let it be $\delta(s_i,x_h)=s_j$ and $\lambda(s_i,x_h)=y_t$.

Transitions of two cases are possible in FSM A^i:

(i) $s_i,s_j\in S^i$ i.e. s_i and s_j are both the states of component FSM A^i, then in A^i must be

$$\delta^i(s_i,x_h)=s_j,\ \lambda^i(s_i,x_h)=y_t$$

(ii) $s_i\in S^i$, $s_j\in S^p$ are states of different component FSMs then transition function δ^i in S^i must be $\delta^i(s_i,x_h)=s_j$, $\lambda^i(s_i,x_h)=y_t\cup\{z_j\}$ and in S^p must be $\delta^p(b_p,z_j)=s_j$, $\lambda^p(b_p,z_j)=y_0$, where y_0 corresponds to the output vector containing only zero values, none of $y_1,\ldots,y_N$ are written in the column $Y(s^t,s^{t`1})$ at the transition (b_p,z_j) in FSM A^p.

5. Initial state s_i of the component FSM A^i: is s_0, if $s_0\in S^i$ or b_i, otherwise.

Thus, described above procedure of FSM decomposition is reduced to

(i) copying the row $s_is_jX(s_i,s_j)Y(s_i,s_j)$ from the table of FSM A to the table of component FSM A^i if s_i and s_j are states of A^i;

(ii) replacement of the row $s_is_jX(s_i,s_j)Y(s_i,s_j)$ of FSM A by the row $s_ib_mX(s_i,s_j)Y(s_i,s_j)z_j$ in the transition table of of FSM A^m and the row $b_pa_jz_j$ in the transition table of FSM A^p, if s_i is a state of A^m and s_j is a state of A^p, $m\neq p$.

As a result in our example (Fig.1) there will be three component FSMs A^1,A^2 and A^3 (Fig. 2, Tables 3,4,5). Let us describe network N by resulting FSM $A_N=(S_N,X_N,Y_N,\delta_N,\lambda_N)$, where $S_N=\{s_1\cdot s_2\cdot \ldots\cdot s_k\}\subseteq T^1\times T^2\times\ldots\times T^k$, if partition π has k blocks.

Each state s_N of resulting FSM A_N is k-tuple $s_1 \cdot s_2 \cdot \ldots \cdot s_k$, where s_i is the state of component FSM A_i. In a time the state of one component FSM A_j coincides with the state of original FSM A, last components are in the "waiting states" $b_1, b_2, \ldots b_{j-1}, b_{j+1}, \ldots, b_k$. Let state s_1 be initial state of FSM A in our example, then state $s_1b_2b_3$ is initial state of resulting FSM A_N. In Fig.3 we can see how states of resulting FSM A_N are changed during of three transitions.

The first transition changes states of the first component A^1 and the second component A^2. FSM A^1 transits into a "waiting state" b_1 and sends the control signal z_3 to the inputs of component FSM A^3 . Control signal transits the FSM A^3 into state s_3. The transition changes the state of the second component FSM A^2 then component FSM A^3 transits into waiting state b_3 and put control signal z_6 onto inputs of A^2. Control signal z_6 transits FSM A^2 into state s_6.

FSM decomposition problem is formulated here as follows: it is necessary to find the decomposition (S,π) of FSM A that the partition $\pi=\{S^1,S^2,\ldots,S^k\}=\{S^i\}$on the

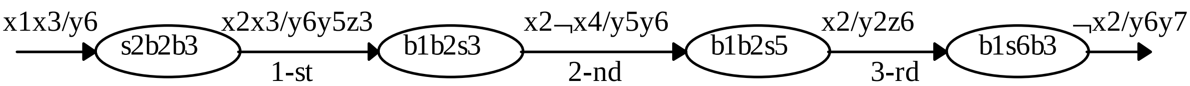

Fig.3. Fragment of FSM A_N Transition Graph

state set S would have a minimal number of blocks and for each component FSM number inputs and/or outputs are restricted by some parameters.

3. **An Entropy-Based Approach for Power Analysis**

The concepts of information theory have proved their usefulness in different domains of digital design. Two new measures based on entropy and information energy are proposed in [1] for estimating of the energy consumption in circuits represented at the gate level of the design. In [2] the method of power estimation capability based on the use of entropy is considered for circuits given only its Boolean functional descriptions.

In our paper we can take an information theory approach for power estimation in a FSM network. With this object in view we construct and investigate an entropical model of a FSM network.

Let us consider the stochastic model $\mathrm{R} = (\mathrm{S}, \mathrm{E}, \mathrm{P}, \mathrm{G})$ of finite state machine A=(X,Y,S,δ,λ), where $G = [g_{ij}]$, $0 \le g_{ij} \le 1$, $\sum_j g_{ij} = 1$ is Transition Probability Matrix that is called Stochastic Matrix. And $\mathrm{P} = [\mathrm{p}_1, \mathrm{p}_2, ..., \mathrm{p}_n]$ is Limiting State Probabilities vector [5]. The entropy of stochastic finite state machine represented by the system $\mathrm{R} = (\mathrm{S}, \mathrm{E}, \mathrm{P}, \mathrm{G})$ is equal to the own information contained in the FSM as per its states.

$$H(S) = |\sum_i p(s_i) I(s_i)| = |\sum_i p(s_i) \log_2 p(s_i)| \qquad (1)$$

In the result of the decomposition of FSM A we loose some information [6] about an interaction of ‘elements’ in original FSM. To coordinate the functions of components in a FSM network it is necessary to increase the summary of input and output channels of the device, i.e., to increase the summary value of transfer information.. These reasons lead to an increase of the entropy value of a FSM network in the comparison with the entropy of the original FSM. So the partition π of the state space S of original FSM into formally independent groups leads to the loss of some information about mutual dependencies between states in a different partition blocks.

Let us consider a FSM network as a complicated stochastic system and propose that the descriptions of component FSMs as stochastic systems are well known, i.e. we know the description of the each component FSM by Markov Chain model. In this case we can calculate the distribution of state probabilities and conditional state probabilities for each component. As a measure of a mutual dependence between components we consider the quantity of the mutual information proposed by Shennon [6]. Entropy of FSM network characterising indefiniteness of states of component FSMs is equal to own information in FSM network concerning its component FSMs. It has the following form:

$$H(N)=H(A^1,A^2,\dots,A^k)=I(A^1,A^2,\dots,A^k)=\sum I(A^i)-\sum I(A^i;A^j)+\sum I(A^i,A^j;A^h)+\dots$$

$$\ldots+(-1)^{k-1}I(A^1,A^2,\ldots,A^k) \qquad (2)$$

where the summation is over all subline´s indexes i<j<h<…<k. It is evidently that the entropy value H(X) is defined by taking into consideration of all mutual dependencies between components. This expression determines lower bound of entropy value in the decomposable system. If all mutual dependencies in decomposable system are ignored then we receive upper bound of entropy value of decomposed system:

$$H(N)=\sum_i I(A^i) = \sum_i H(A^i) \qquad (3)$$

Thus, in this case component FSMs can be regarded as formally independent parameters. As appears from the above (look Tables 1-3, Fig.1), the transition graphs of the original FSM A is isomorphic to the resulting FSM A_N. therefore the vector $P=[p_1,p_2,...,p_n]$ of limiting probabilities of FSM A coincides with the vector of limiting probabilities of resulting FSM A_N

Let FSM network N be designed by above considered method and N has k component FSMs. Assume that $s_N \in S_N$, where $s_N= b_1 \cdot b_2 \cdot \ldots b_{j-1} \cdot\ s_i \cdot\ b_{j+1} \cdot\ \ldots \cdot\ b_k$. Evidently that

$$p(s_N) =p(b_1)\cdot p(b_2)\cdot \ldots p(b_{j-1})\cdot\ p(s_i)\cdot\ p(b_{j+1})\cdot\ \ldots \cdot\ p(b_k)$$

Conclusion 1. The global probability $p(s_N)$ of state $s_N= b_1 \cdot b_2 \cdot \ldots b_{j-1} \cdot\ s_i \cdot\ b_{j+1} \cdot\ \ldots \cdot\ b_k$ is equal to global probability $p(s_i)$ of state s_i . It follows that $p(b_1)=\cdot p(b_2)= \ldots =p(b_{j-1})= p(b_{j+1})=\ldots= p(b_k)=1$.

Conclusion 2. The entropy value in FSM network N constructed by the above considered method is defined by the expression (2).

4. Design Methodology and Experimental Results

As a first step toward a high-level power estimation capability, we have implemented a technique for estimation of the average switching activity of a FSM network based on the FSM network entropy measure. FSM networks are designed from benchmark FSMs represented in kis format. We implemented two approaches for the entropy estimation. The first one is based on the entropy calculation using expression (4), second one is based on the entropy estimation presented by the expression (3). The technique was tested on a number of MCNC FSMs. Results of two different experiments are presented in Table 1. First we estimate the value of entropy H(A) before the FSM decomposition then FSM A is decomposed into FSM networks making use of three decomposition methods. Decomposition is implemented in the proposition that the number k of component FSMs has been given a priori. The first solution determines the FSM network in which the number of inputs and outputs of component FSM are minimised simultaneously. The second one minimises only the number of component inputs and the third one minimises the number of component outputs. The results of experiments are demonstrated in Table 4 where column $H(A^i)$ determines values of the entropy of component FSMs. Column H(N) is the entropy of network N described equation (3). Column marked by % gives the increase of the entropy value in the result of a FSM decomposition.

N	FSM	H(A)	n	k	$H(A^i)$	H(N)	%
1	pak	3.2516	1	3	(2.2),(2.31),(2.21)	6.72	106
			2	3	(2.12),(1.56),(2.21)	5.97	83.7
			3	3	(2.2),(2.31),(2.21)	6.72	106
2	bbara	2.6627	1	3	(1.72), (1.9), (2.16)	5.78	117
			2	3	(2.09), (1.9), (1.9)	5.89	121
			3	3	(1.72),(1.87),(2.04)	5.63	111
3	sand	4.5928	1	3	(3.06),(3.2),(3.25)	9.52	107
			2	3	(3.04),(3.25),(3.25)	9.54	108
			3	3	(2.84), (3.3) (3.3)	9.44	106
4	sse	2.3962	1	3	(1.5),(2),(2.1)	5.84	143
			2	3	(2), (1.61),(1.78)	5.395	125
			3	3	(1.5),(2),(2.1)	5.84	143
5	planet	5.2568	1	2	(4.4),(4.02)	8.417	60.1
			1	2	(4.4),(4.02)	8.417	60.1
			2	2	(4.38), (4.04)	8.416	60

Table 4. Entropy Evaluation in FSM networks

Conclusions

An information theory approach for entropy evaluation in FSM networks have been proposed in our paper. Two models for entropy estimation are considered here. The first one accounts for the summary entropy of separate component FSMs. It is an upper bound of the entropy value in a FSM network. The second model that accounts for an information exchange between components gives more exact value the entropy of a FSM network. This model is used as a measure of power dissipation in combinational circuits of network components. The algorithms of a FSM decomposition and the entropy power estimation for upper bound in FSM networks were programmed and tested at FSM benchmarks.

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